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THE INFLUENCE OF PLASTIC DEFORMATION ON  
THE TIME-DECREASE OF PERMEABILITY IN TRANSFORMER STEEL

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The paper gives the results of measurements of the influence of plastic deformations in the time-decrease of permeability in hot rolled transformer steel. In the first part measurements were performed on samples rolled in a mill. The measurements showed a drop of time decrease of permeability at large elongations and a maximum at small elongations. In the second part measurements were performed on samples stretched in a tensile testing machine. The results of measurements are characterized by typical wavy shape of time-decrease of permeability. The obtained results are connected with the variations of lengths of the paths of impurities diffusing in the crystal lattice.

The influence of plastic deformation  
on the time - decrease of permeability in  
transformer steel

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## 1. Introduction

The purpose of this paper is to present some results of measurements of the influence of plastic deformation on the time-decrease of permeability in hot rolled transformer steel.

The former papers on time-decrease of permeability have shown that this effect appears in a high degree in cores stamped out from hot rolled transformer sheets (approx. 4% Si,  $\mu_{20} \geq 700$  Gs/Oe)<sup>1,3,4</sup>. Time-decrease of permeability at low fields is in the range of 24 hours and the relative change of permeability reaches 30%. The annealing of such cores performed at low temperatures (about 800°C) lowers the time of decrease of permeability to 7 hours approximately. But the annealing process in high temperatures (about 1250°C - in hydrogen) practically erases this effect reducing the time of decrease of permeability to a few minutes only<sup>2,3,4</sup>.

It is known from the basic research on the magnetic after-effect in general and on time-decrease of permeability in particular, that the essential cause of these effects is the diffusion of impurity ions in the crystal lattice of ferromagnetic material<sup>5,6</sup>. The technical ferromagnetic materials such as silicon steel contain-as a result of technological process-impurities which make the magnetic properties worse and cause the after-effect.

The internal stresses introduced by the impurity strains are one of the causes changing the magnetic properties, as the decrease of permeability for instance. External stresses introduced by the external forces as well as their permanent

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consequences in the form of plastic deformations, which remain after the removal of these forces, produce the change of magnetic properties in a similar way. The plastic deformations are often encountered in practical applications of magnetic materials. The deformation at the edges of stampings cut out from the transformer sheets is one typical form.

The stampings with heavy deformations at the edges show a high degree of time-decrease of permeability reaching 24 hours. The deformations of this type appear frequently in transformer cores in cases where annealing process does not follow. Therefore it is essential to explain to what degree they affect the time-decrease of permeability.

## 2. The methods of introducing the plastic deformations

As it was mentioned before the stampings cut out from sheets of hot rolled silicon steel contain the plastic deformations which chiefly appear at the edges of stampings. The material inside is rather not deformed and thus the tests reported in earlier publications had not treated the problem quite correctly<sup>1,2,3,4</sup>.

It appears that the introduction of the deformations in the whole bulk of tested material is necessary in order to check the influence of plastic deformations on the time-decrease of permeability adequately.

Two methods of introduction of such deformations were used. The first one is that of stretching the samples (about 30 cm long and about 5 cm wide) at a tensile testing machine, while

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the second consists of rolling them at a mill. However, the resulting character of plastic deformation obtained is not the same in both cases, although in both cases the elongation of samples occurs.

The introduction of plastic deformations by rolling enables setting them at any angle to the axis of the sample along which the magnetic field is applied at the time of testing.

### 3. The methods of measurements

The samples thus obtained-demagnetized and left for a period of time in which the permeability reached a constant value-were put into a yoke. The tested sample was surrounded by two windings which enabled the use of the transformer and bridge methods of permeability measurements in the range of low fields.

The measurements were carried out at the field intensity  $\bar{H} = 20 \text{ mOe}$ . usually applied to measure the magnetic properties of low fields<sup>8</sup>. Such a high value of the field intensity, which is essential in given conditions to obtain sufficiently high accuracy of measurements, obscures on the other hand to a certain degree the investigated phenomenon.

Prior to the proper measurements of permeability as function of time the yoke together with samples was demagnetized by means of strong A.C. field gradually decreasing to zero.

The value of time after which the steady value of permeability was first obtained was defined as the time-decrease of permeability  $t_d$ .

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The measurements were carried out at the room temperature (about 18°C).

The transformer method of permeability measurements is very convenient in cases when time of measurements is not too long (does not exceed one hour). During such a period of time the constant voltage and frequency of a generator may be easily maintained. Because of the errors of indication this method cannot assure the detection of a very small change of permeability (lower than few per cent) which occur in a relatively long period of time-up to about 24 hours. Such permeability changes can however be detected by means of a bridge method. This method on the other hand is difficult in the case of measurements performed immediately after demagnetization, since the time of bridge balance is long (several seconds and more) especially at the beginning of the measurements.

We notice from the above discussion that the obtained value of the time-decrease of permeability can depend on the method of test. When applying the transformer method one should expect lower values of the time-decrease of permeability.

We should also emphasize the fact that another value in which we are interested, namely the total increase of reluctivity

$$\tau_1 = \Delta\left(\frac{1}{\mu}\right) = \left(\frac{1}{\mu_\infty} - \frac{1}{\mu'}\right) \quad (1)$$

depends to a considerable extent on the first measurable value of permeability  $\mu^*$ . The normalization of the starting time of measurement of  $\mu^*$  is essential for comparison of results

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of the two methods, since in transformer method the measurement points are obtained earlier than those in the bridge method. In the measurements cited here we assume that  $\mu'$  is determined at 30 seconds.

#### 4. The tested materials.

Most of our measurements pertain to the silicon steel in the shape of hot-rolled factory annealed sheets containing  $4 \pm 4.5\%$  of silicon and having a total loss value  $P_{10} = 1.05 - 1.3$  W/kg and permeability value  $\mu_{20} \gg 700$  Gs/Oe.

The influence of field intensity on the permeability of this material is shown in Fig. 1. The permeability after demagnetization as a function of time is shown by the curve a in Fig. 2. The permeability has been measured here by means of the bridge method.

From the curves shown we notice that a considerable effect of time-decrease of permeability is associated with the characteristic lower bend of the function  $\mu = \mu(\bar{H})$  in agreement with Feldtkeller's paper<sup>9</sup>.

Certain tests were also performed using samples annealed in hydrogen at temperature of  $1250^\circ\text{C}$  during 4 hours. The time decrease of permeability of such samples decreased to approximately 10 minutes (curve b - Fig. 2) while the permeability value in the steady state increased generally up to more than 1000 Gs/Oe (Table I pos. 2).

The carbon content in the annealed samples was in the range of 0.02% while in the unannealed samples was a little more (0.026-0.030%).



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5. Testing of the rolled samples

The influence of plastic deformations, introduced by rolling the samples on their time-decrease of permeability  $t_d$  and on the total increase of reluctance  $r_1$  was tested in the first part of this work (Fig. 3). Measurements performed on samples cut from sheets have shown the characteristic and typical drop of the time-decrease of permeability for higher values of relative elongations in accordance with the empirical formula:

$$t_d = a \left( \frac{\Delta l}{l} \right)^{-b} \quad (2)$$

Average coefficients  $a$  and  $b$  for the tested materials are:

$$a \approx 50 \div 70 \text{ (sec)}$$

$$b \approx 1.0 \div 1.5$$

The above figures apply both to the measurements performed by means of transformer method and by bridge method.

For small values of relative elongations the maximum of the time-decrease of permeability  $t_d$  appears in the region of  $0.1 \div 0.2\%$  (Fig. 4).

Although the transformer method of measurements shows the lower values of the time-decrease of permeability than those obtained by the bridge method, the position of maximum is in both cases the same.

Similar results are obtained for the samples annealed in hydrogen at high temperature.

In Fig. 3 and 4 the functions  $t_f = f_1 \left( \frac{\Delta l}{l} \right)$  and  $r_1 = f_2 \left( \frac{\Delta l}{l} \right)$  were given simultaneously. It appears from these figures that

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the total increase of reluctance  $r_1$  changes in a similar way as the time-decrease of permeability  $t_d$ .

#### 6. Testing of the stretched samples

In the second part of this paper we shall discuss the influence of plastic deformations introduced by stretching the samples on the time decrease of permeability  $t_d$  and on the total increase of reluctance  $r_1$ . Measurements were performed on the same materials as described before with the difference that in this part the tests were done on the same samples strained consecutively to the breaking point.

In Fig. 5 the results of the following measurements are given: The time decrease of permeability  $t_d$ , the total increase of reluctance  $r_1$  and the relative elongation  $\frac{\Delta l}{l}$  as functions of the stress producing the elongation. In Fig. 6 these results are shown for the sample annealed in hydrogen in high temperature but having lowered permeability. The fact that in certain cases the permeability of hot-rolled transformer steel decreases considerably after such an annealing is worth mention<sup>10</sup>.

The results of measurements shown in these figures are characterized by the typical wavy shape of the time-decrease of permeability  $t_d$  and the total increase of reluctance  $r_1$  as function of stress.

We notice, that the time decrease of permeability as well as the total increase of reluctance show simultaneous increase and decrease as in the case of rolled samples. We can also state,

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that just before breaking of the sample the tested quantities decrease.

It should be emphasized that the results of measurements given in Fig. 5 and 6 are obtained by means of transformer method. Comparative measurements performed by means of bridge method showed similar periodical changes of the tested quantities, but differed in absolute values obtained.

## 2. Discussion of results

From the described measurements it appears that the plastic deformations exert a significant influence on the time-decrease of permeability. This effect depends not only on the magnitude of deformations but also on their character. The time-decrease of permeability corresponding to high values of elongations obtained by rolling decreases considerably and at low values of elongations exhibits a maximum. Plastic deformations obtained by stretching the samples affect the tested phenomenon in somewhat different manner. The time-decrease of permeability undergoes periodical changes and reaches values both higher and lower than the primary value (deformations excluded).

As it was already mentioned, the total increase of reluctance  $R_1$  changes in a manner essentially similar to that of the time-decrease of permeability. It should be observed that the total increase of reluctance depends not only on the magnitude of time-decrease of permeability, but also on the value of permeability in the steady state. The stresses introduced by plastic

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deformations have a considerably influence on the steady-state permeability. When small elongations are obtained by rolling, increased permeability is observed in steady state (Fig. 4). This effect is caused by positive magnetostriction of silicon steel. It appears periodically in stretched samples.

In Tables I, II and III the results of time functions analysis according to Rathenau are presented<sup>11</sup>. These data show that the constants of relaxation increase and decrease similarly to the total time-decrease of permeability. In the unannealed samples the amplitude of second relaxation  $P_{12}$  predominates generally. This quantity almost disappears after annealing of samples and appears again after introduction of adequate deformations. We may conclude that the plastic deformations in certain conditions weaken and in the others emphasize the effect of time-decrease of permeability. This results undoubtedly from variations of lengths of the paths of impurities diffusing in the crystal lattice of tested material. These paths depend however on the degree and on the manner of the lattice deformation.

Silicon Steel /about 4,5% Si/, total core loss 1,05 W/kg  
/Bridge method/

No.	The state of material	$\sigma_{20}$ kg/cm <sup>2</sup>	$r_1 \cdot 10^4$	$t_1$ min	$\tau_1$ min	$\tau_2$ min	$r_{11} \cdot 10^4$	$r_{12} \cdot 10^4$	Remarks
1.	Unannealed sample	705	5	150	40	8	2,2	2,6	Sample no 3 measured in the yoke
2.	Sample annealed in hydrogen	1150	1,7	8	4,3	-	1,6	—	"
3.	As <sub>2</sub> stretched $\sigma = 27,2 \text{ kg/mm}^2$ , $\frac{\Delta l}{l} \approx 0,3\%$	930	2,8	25	14	1,7	1,4	1,2	"
4.	As <sub>2</sub> stretched $\sigma = 37 \text{ kg/mm}^2$ , $\frac{\Delta l}{l} \approx 0,9\%$	190	17,6	140	36	3	11,4	3,7	Sample no 6 twisted in to a ring.

T A B L E    I I  
Silicon Steel /about 4,5% Si/ total core loss 1,05 W/kg  
/ Bridge method /

No.	The state of material	$\mu_{20}$ Ga/00	$P_1 \cdot 10^4$	$G_{20}$ ga	$T_1$ min	$T_2$ min	$P_{11} \cdot 10^4$	$P_{12} \cdot 10^4$	REMARKS
1.	Unannealed sample	795	1,9	140	114	13	0,3	0,8	Sample no.9 measured in the yoke
2.	Sample annealed in hydrogen	775	1,3	8	3,5	-	1,3	-	"
3.	As rolled $\frac{A_1}{I} \approx 0,6\%$	390	0,38	5	2,5	-	0,34	-	"

**TABLE III**  
**Silicon Steel /about 4,5% Si/ total core loss 1,05 W/kg**  
**(Transformer method )**

No.	The state of material	$A_{20}$ Ga/Gs	$\Sigma_1 \cdot 10^4$	$t_d$ min	$\tau_1$ min	$\tau_2$ min	$\Sigma_{11} \cdot 10^4$	$\Sigma_{12} \cdot 10^4$	Remarks
1.	Unannealed sample	1160	0,55	16	6	-	0,55	-	Sample no 422 measured in the y fig. 2
2.	As 1, stretched $\sigma = 12 \text{ kg/mm}^2, \frac{\Delta l}{l} \approx 0,05\%$	1210	0,13	3	2,2	-	0,0	-	"
3.	As 1, stretched $\sigma = 20 \text{ kg/mm}^2, \frac{\Delta l}{l} \approx 0,15\%$	1075	1,45	60	32,2	3	0,95	0,22	"

## REFERENCES

- 1 A.Smoliński i M.Żbikowski : „Opóźnienia magnetyczne w stalach krzemowych”.  
Archiwum Elektrotechniki 1955, No.3, p.493.
- 2 A.Smoliński i M.Żbikowski : „Opóźnienia magnetyczne w stalach krzemowych poddanych obróbce cieplnej”.  
Archiwum Elektrotechniki 1957, No.2, p.300.
- 3 A.Smoliński i M.Żbikowski : „Opóźnienia magnetyczne w stalach krzemowych gorąco walcowanych”  
~~Referat na Konferencję Naukową Techniczną na temat Blach Elektrotechnicznych, Katowice luty 1957 r.~~  
Prace Instytutu Tele i Radiotechnicznego 1957, No.3, p.41.
- 4 A.Smoliński i M.Żbikowski : „The time - decrease of permeability in transformer steel” Proc.I.E.E. Part III Monograph No.313M, October 1958.
- 5 J.L. Snoek: „Time effects in magnetization”  
Physica 1938, vol 5, No.5, p.663.
- 6 J.L. Snoek: „Magnetic Aftereffect and Chemical Constitution”  
Physica 1939 vol.6, No.2, p.161.
- 7 E.S.Anolick : „The effect of aging on the time decay of permeability in 3% silicon iron”  
Conference on Magnetism and Magnetic Materials Boston 1956, p.438
- 8 DIN E 4 1301: „Weicheisen für Uebertragerblache , 1943 .
- 9 R. Feldtkeller u.G.Sorger : „Magnetische Nachwirkung der Anfangspermeabilität und der Barkhausensprünge”  
A.E.Ü 1953, No.2, p.79.
- 10 A.Smoliński i A.Zawadzki: „ Uzyskiwanie rdzeni klasy C z blachy o zawartości około 4% krzemu  
Dokumentacja techniczna PIT, 1955.



11. G.W. Rathenau: "Time decrease of permeability in iron"

J. Appl. Phys. 1958, No. 3, p. 239.

1. Final values of permeability plotted against field intensity (Bridge method)
2. Variations of permeability after demagnetization (Bridge method)
3. Influence of elongation due to rolling on the time decrease of permeability  $t_d$  and the total increase of reluctivity  $r_1$  (Large elongations, bridge method)
4. Influence of elongation due to rolling on the time decrease of permeability  $t_d$  and the total increase of reluctivity  $r_1$  (Small elongations, bridge method)
5. Influence of ~~pulling~~ stresses on constant values of permeability  $\mu_{20}$ , time decrease of permeability  $t_d$ , total increase of reluctance  $r_1$  and relative elongation  $\frac{\Delta l}{l}$ . Samples unannealed, (Transformer method) and stretched
6. Influence of ~~pulling~~ stresses on constant values of permeability  $\mu_{20}$ , time decrease of permeability  $t_d$ , total increase of reluctance  $r_1$  and relative elongation  $\frac{\Delta l}{l}$ . Samples annealed (but with lowered permeability), (Transformer method).

and stretched

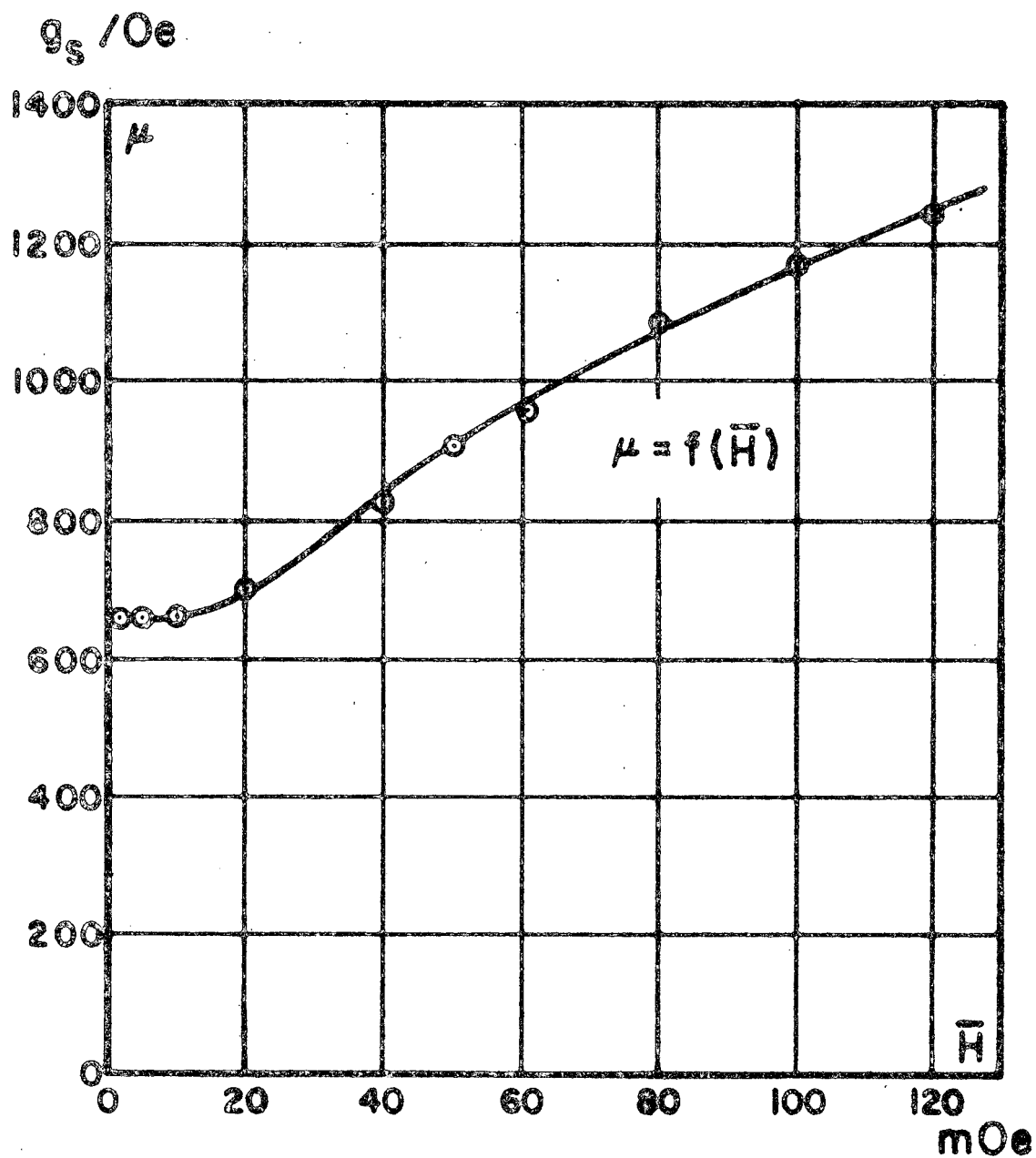


FIG. 1

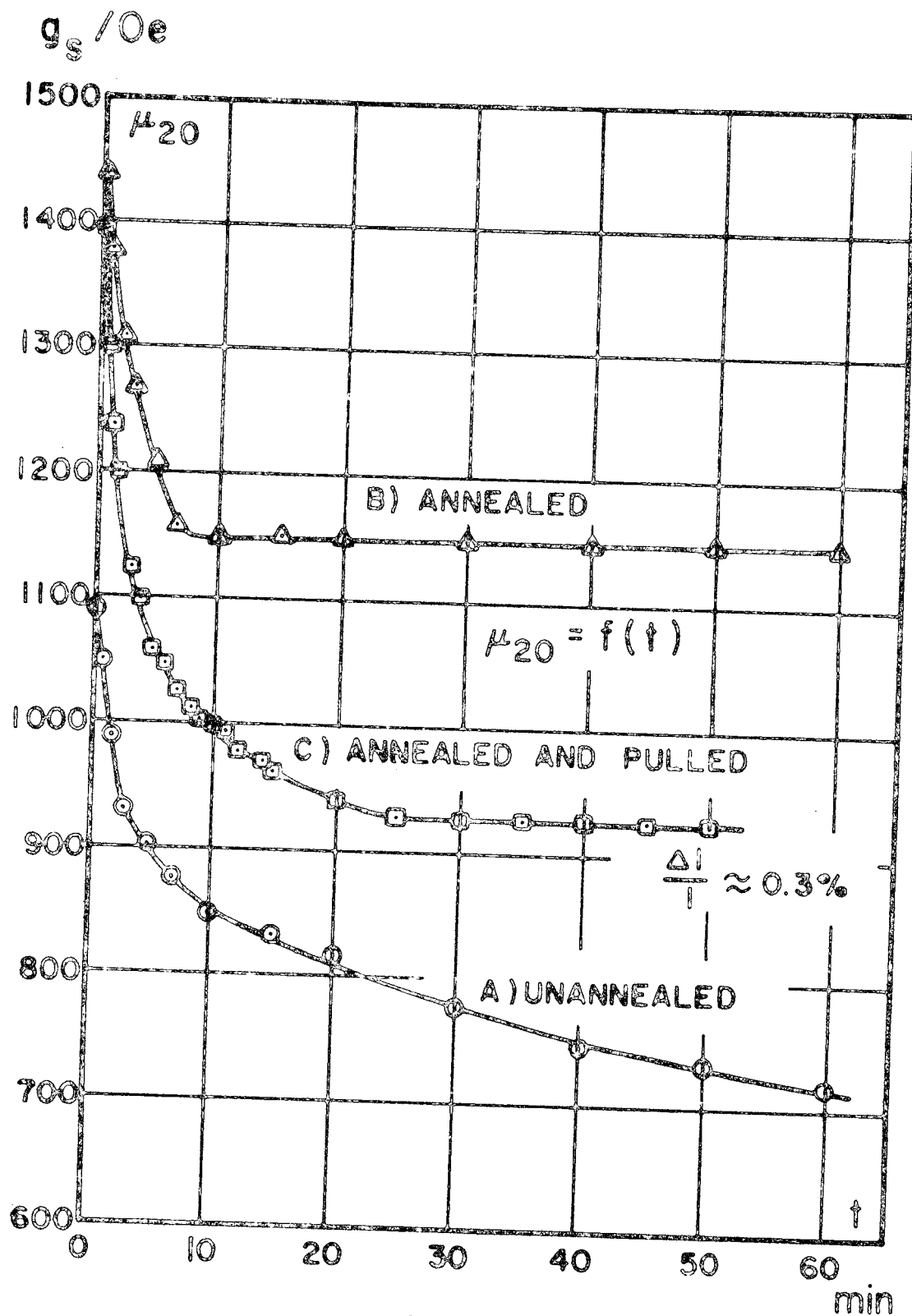


FIG. 2

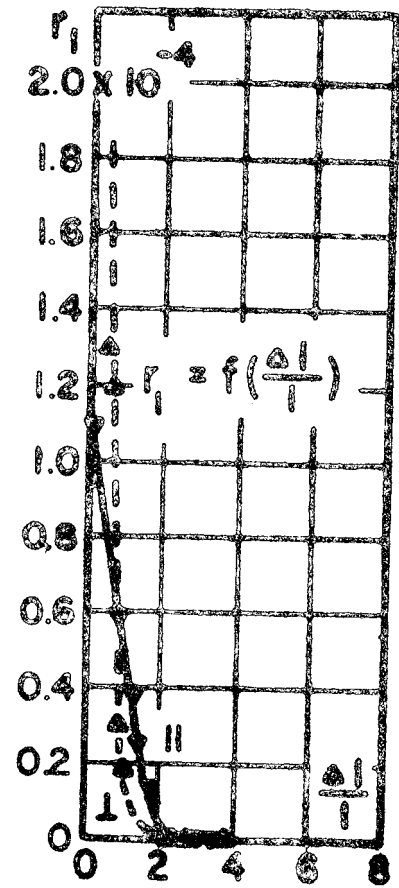
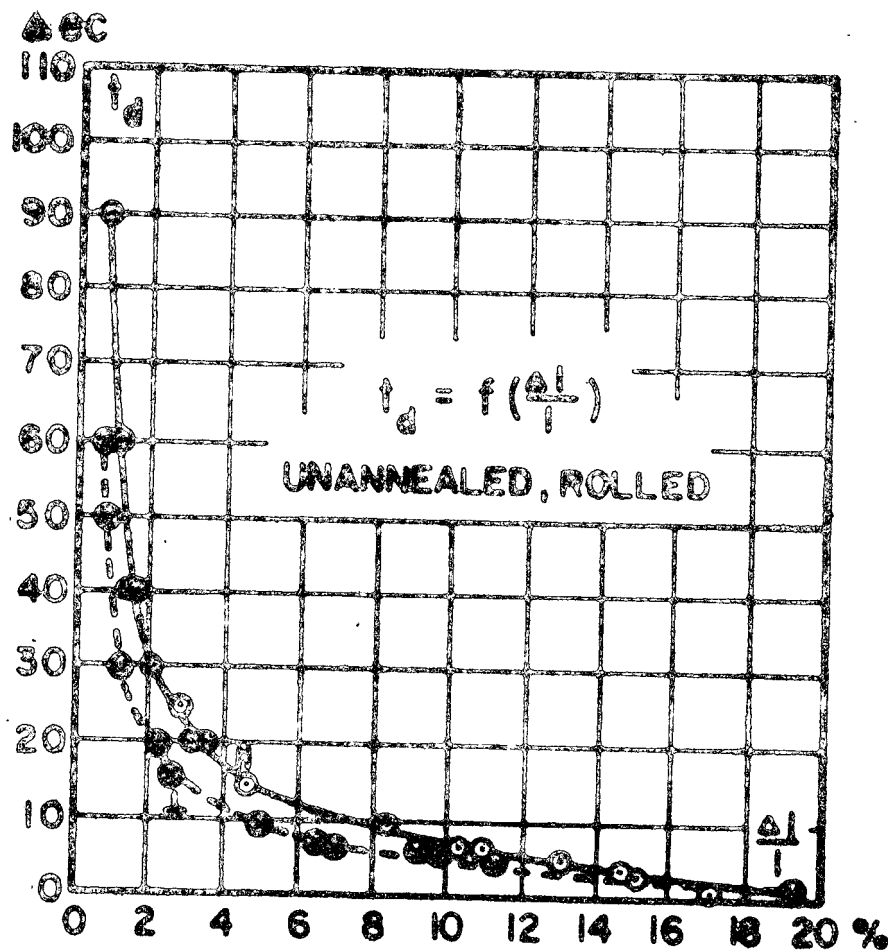


FIG. 3

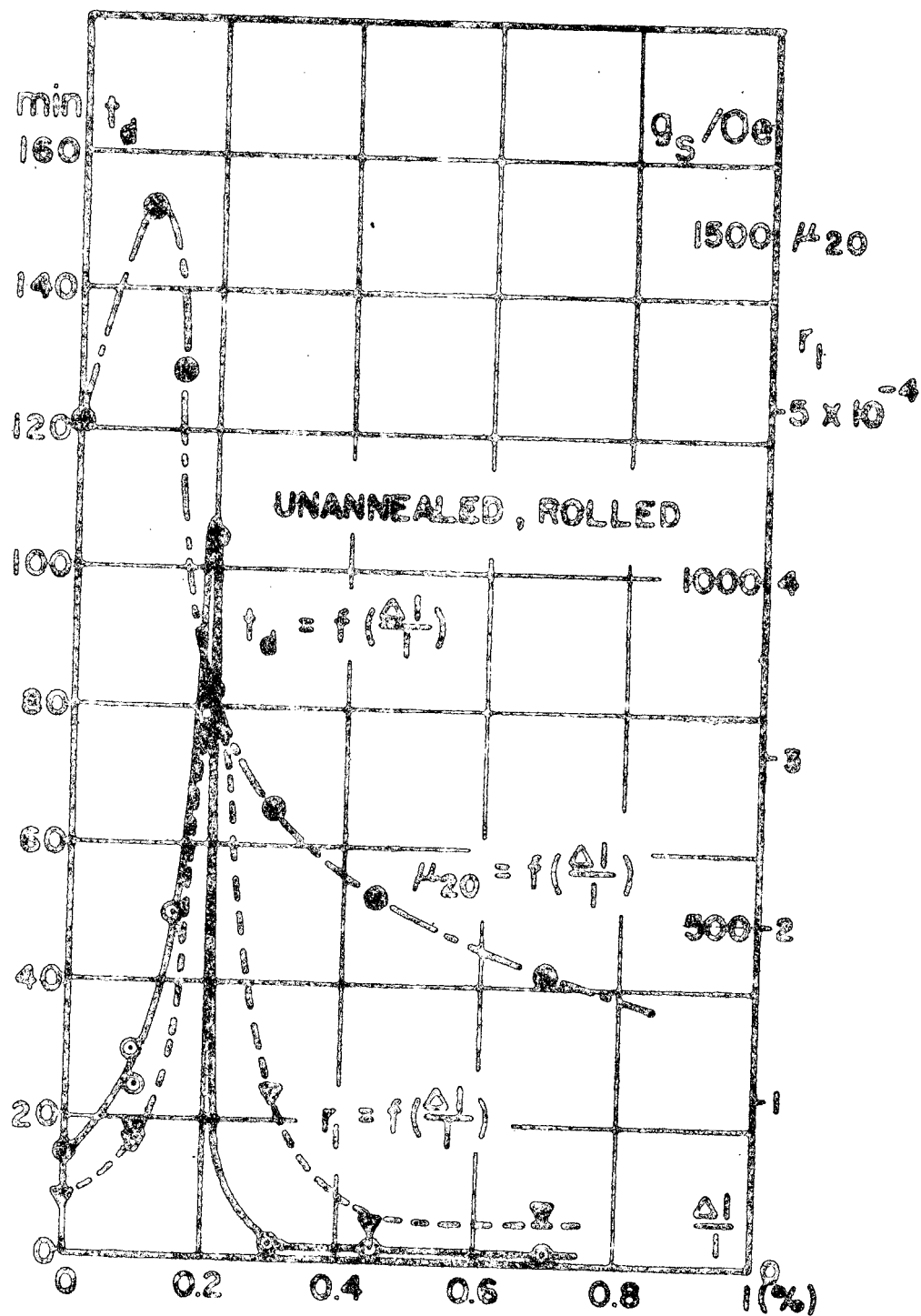


FIG. 4

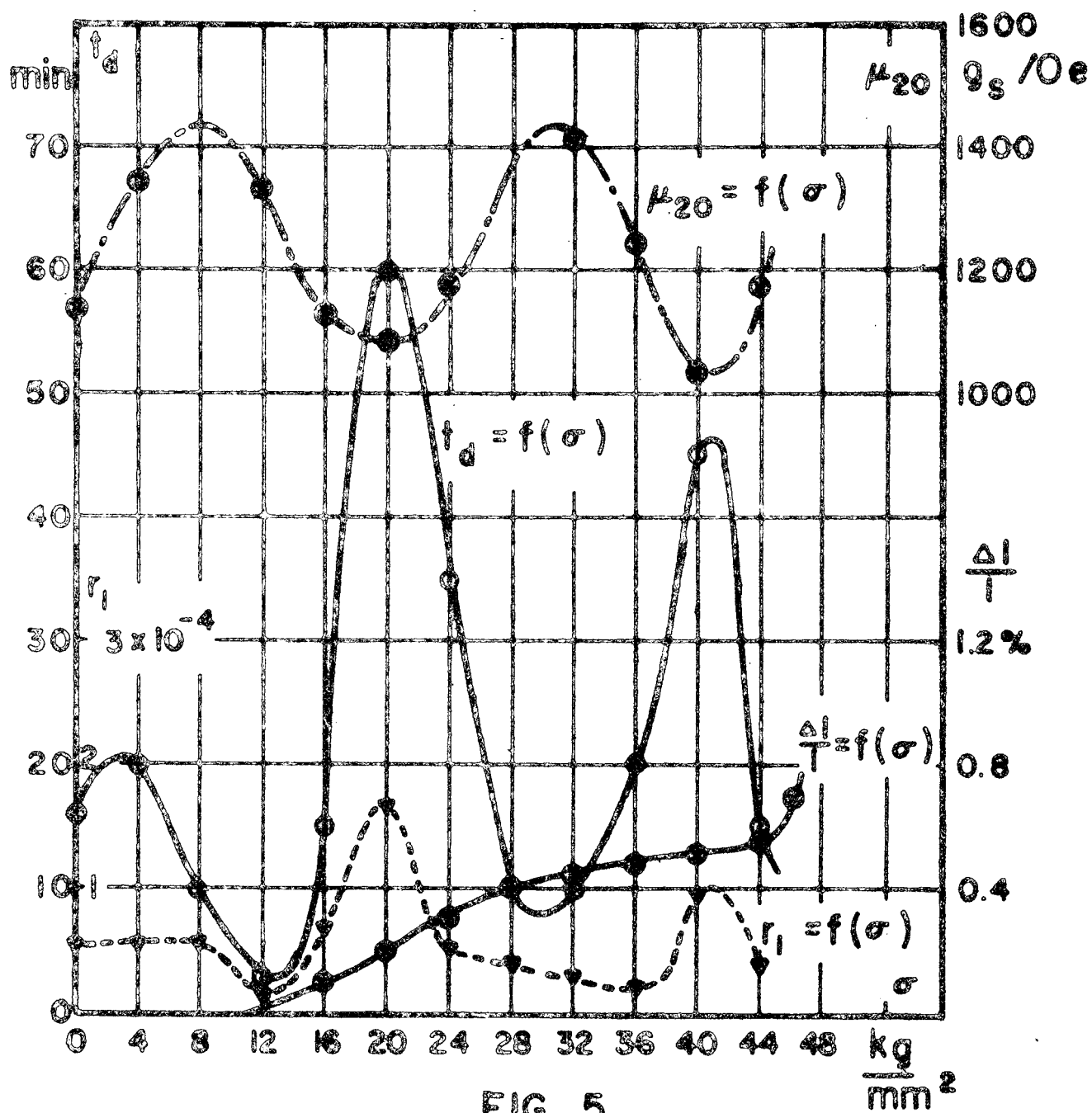


FIG. 5

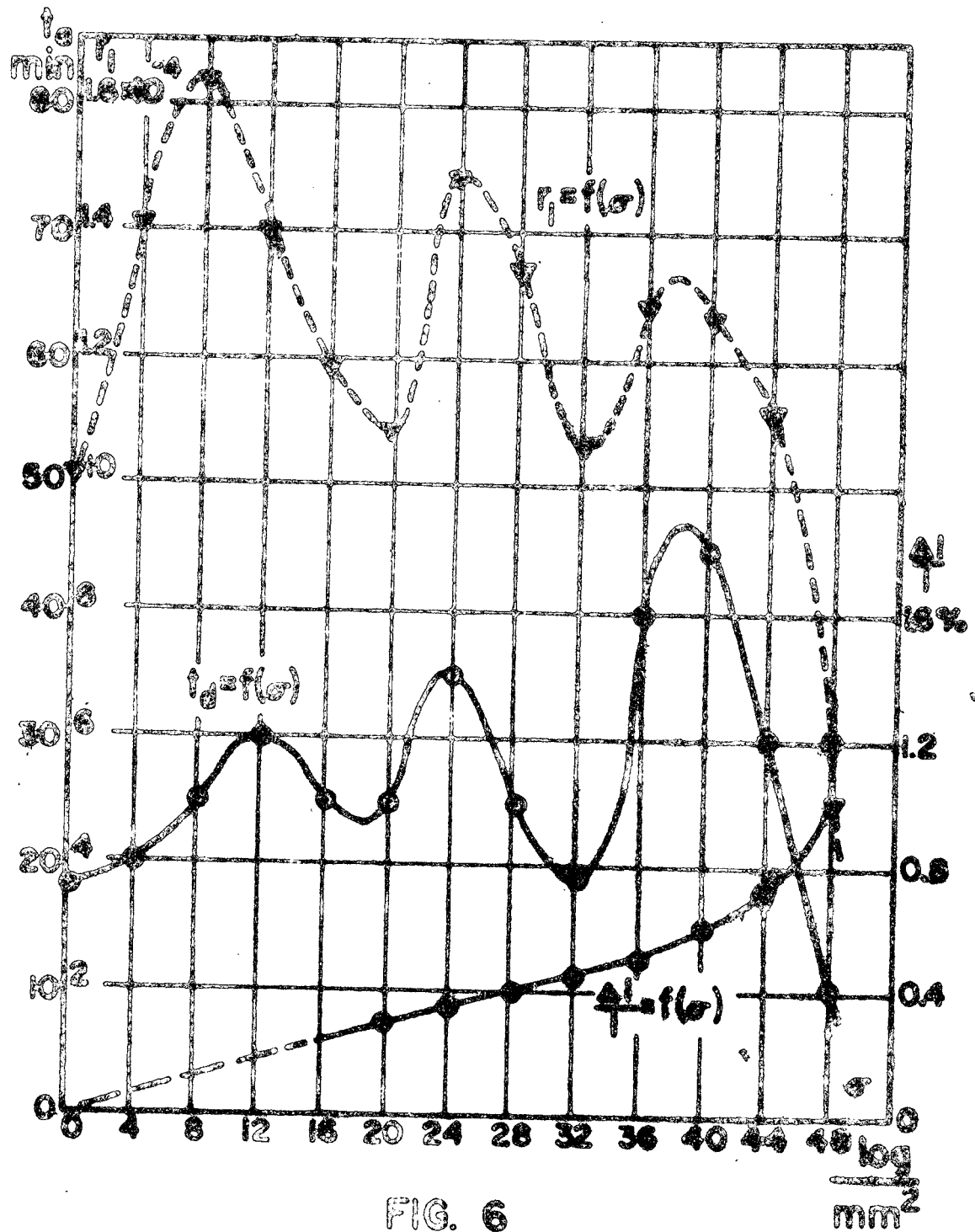


FIG. 6



SOME PROBLEMS OF PHENOMENOLOGICAL THEORY OF FERRO- AND  
ANTIFERROMAGNETISM.

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This paper presents<sup>a</sup> general review of<sup>the</sup> modern state of phenomenological theory of ferro- and antiferromagnetism and the results of some investigations of the authors<sup>b</sup> in this field.

I

In the quantum theory of ferro- and antiferromagnetism there are essential difficulties when attempting to build ~~the~~ adequate microscopic models. To receive quantitative or yet only qualitative results one must make many poorly grounded simplifications, which lessen the value of the results of the theory to a great extent. Therefore it is very useful to try to avoid these difficulties and to create the general phenomenological theory in which there would be<sup>a</sup> minimum of all unproved model approximations. This would give a good chance to formulate those questions, that have real physical meaning.

Weiss<sup>/1/</sup> was the first who long ago used this method in creating his theory of ferromagnetism. This method was further developed in the works of Van Vleck<sup>/2/</sup> and Néel<sup>/3/</sup>, who created the theory of molecular field for antiferro- and ferrimagnetism. But the phenomenological theory of molecular field has only a limited application (high temperatures). The physical nature of molecular field was unknown when creating this theory. Therefore it is natural that the physicists attempted to set up the atomic theory of ferromagnetism. Ising's classical atomic model<sup>/4/</sup>, which played a great role in the development of the microscopic treatment of ferromagnetism should be mentioned here. The fact that the nature of<sup>the</sup> interactions between the atomic magnetic moments still remaind<sup>a</sup> unknown was the defect of this

model as well as of the theory of Weiss. The physical meaning of this exchange interaction was understood only after works of Frenkel<sup>/5/</sup> and Heisenberg<sup>/6/</sup>.

The principal approximation of the molecular field theory consists in taking into account only the long range magnetic order. Therefore it is only natural that attempts<sup>t/</sup> to improve the Weiss's theory were made, considering the short range magnetic order. The necessity of this improvement resulted from the short range character of exchange interaction also.

A number of papers (of Peierls<sup>/8/</sup> and others<sup>/9/</sup>) in which the short range correlation in spin systems were taken into account using different methods were published. But in fact these "improved" treatments in some respects worsened the principal positions of the theory due to a greater number of rough model approximations. However the results of molecular field theory in spite of rough character of approximations, gave good qualitative explanation of ferromagnetic properties of matter. The solution of this paradox was possible after appearance of exact thermodynamic treatment of ferromagnetism.

The fundamental works of Landau<sup>/10/</sup> on the theory of phase transitions laid the foundation for the beginning of creation of the consistent thermodynamical theory of condensed mediums. His method permitted to make a base for all the correct results of Weiss's theory, and to build the exact thermodynamical theory of magnetic properties of matter<sup>x/</sup>.

<sup>x/</sup> One of the authors<sup>/11/</sup> and Ginsburg<sup>/12/</sup> first applied this method to ferromagnetic phase transitions. The Landau's method can be extended<sup>/11, 15/</sup> for the case of magnetic short range order with the help of statistical thermodynamics theory of the fluctuations. The work of Zaitzev<sup>/17/</sup> must be mentioned in connection with this problem also.

The Landau's method consists in using the most general ideas of group theory. The explicit form of the expansion of thermodynamic potential (in series by the power of small parameter) near the phase transition points <sup>is obtained</sup> ~~had been received~~ using the general symmetry considerations. The magnetic state equation can be found from the general conditions of thermodynamic equilibrium. The recent work of Dzialoshinsky<sup>/13/</sup> can be mentioned as an example of great possibilities of this method. The physical explanation of "weak" ferromagnetism, for a long time known in  $\text{Fe}_2\text{O}_3$  and recently found in a number of other compounds, was given in that work. But the thermodynamic theory also possesses some defects. Firstly, it contains some phenomenological constants which require detailed information about atomic structure of crystals. Secondly, the results of this theory can be obtained only in the narrow temperature interval near the phase transition points. This results from the fact that only in this temperature range we can be satisfied with the few first members in the expansion of thermodynamic potential by the powers of small parameters (magnetization). Therefore it was necessary to develop the theory to be applied for the low temperature range. The wave functions and energy spectrum of crystal electrons must be determined for complete microscopic description of these substances. However this problem cannot be solved practically. There is one important reason why the macroscopic bodies cannot be in precisely stationary states besides the mathematical difficulties due to great many degrees of freedom. The fact is that the breadth of energy levels, resulting from their quantum nature ( $\Delta E \sim \frac{\hbar}{\Delta t}$ ) is much greater than the splitting of these levels. Therefore, it is necessary to use the quantumstatistical treatment with the density matrix, for precise description of macrobodies. This method was realized in the above mentioned work of Heisenberg<sup>/6/</sup>. He substituted the true energy spectrum of crystal electrons by

statistical mean values - the energetic centers of gravity. However this method is a very rough approximation of quantum statistical treatment. It can be used only for description of the properties of magnetism only in the region near (or more exactly, above) the Curie and Néel points, like in the method of molecular fields. Therefore, it is necessary to develop a consistent quantum statistical theory for the wide range of temperatures including the very low ones. However there is no such theory at present. But it is possible to mention cases when we can use the quantum mechanical theory for macrobodies. This may be done when calculating the ground state (the lowest energy state) and the nearest, weakly excited states. For the latter case ~~the~~ the motion of complex macro systems may be described with high precision as the free gas of quasiparticles or elementary excitations. It is easy to find the energy spectrum and wave functions for this gas.

The method of quasiparticles was used in microscopic ferromagnetic models ~~possess~~ with great success. But these models possess some defects due to rough simplifications. In 1935 Landau and Lifshits<sup>/18/</sup> gave the program for the generalization of the phenomenological theory of ferromagnetism for the range of low temperatures. Herring and Kittel<sup>/19/</sup> were the first to use this method for creation of the phenomenological theory of spin waves (ferromagnons)<sup>x/</sup>.

## 2.

Ferro-, antiferro-, and ferrimagnetics differ from all other substances by their atomic magnetic structure. This means, that magneto-active ions in crystals of these substances possess nonzero time average values of spin or orbit magnetic moments ( $\langle \vec{S}_i \rangle_t$ ). All these moments in ferromagnetics are parallel. Therefore a spontaneous resulting moment appears in these crystals. In antiferromagnetics we have

<sup>x/</sup> Also see the works of Heller and Kramers<sup>/20A/</sup> and Döring<sup>/20B/</sup>.

the antiparallel arrangement of moments with complete compensation in each elementary cell. This compensation is absent in ferrimagnetics and therefore in these crystals nonzero spontaneous difference resulting moment exists. And lastly in paramagnetics the time average moment in all lattice points are equal to zero<sup>x/</sup>. The space distribution of  $\langle \vec{S}_j \rangle_t$  can be given as  $\delta$ -function

$$\sum_j \langle \vec{S}_j \rangle_t \delta(\vec{r} - \vec{r}_j) \quad (I)$$

In equation (I) the sum is taken over all lattice points.  $\delta$ -function is the analytic expression of discrete nature of crystals with magnetic atomic structure. But this kind of description is not always necessary as can be seen from the Debye's heat capacity theory. The crystal can be regarded as a quasicontinuous medium, if we take into account only such modes of motion, in which the discrete lattice structure is not important. This forms the basis for the phenomenological description of magnetics. The term "quasi" emphasizes the fact that we have the medium with the important properties of discrete crystal atomic structure (crystal symmetry elements, differences in types of lattice points and in types of magnetoactive ions and so on).

The oscillating character of motions must be taken into account when selecting the criterion for those degrees of freedom of crystal at which the lattice behaves as quasicontinuum. If the characteristic wavelength of this mode of motions is much greater than the lattice constant the crystal can be considered as quasicontinuum. A large number of atomic particles simultaneously take part in these oscillations. Therefore they are of co-operative character. The transition from discrete to continuous description is expressed mathematically by placing the magnetic moment density functions  $\vec{M}_j(\vec{r})$  instead of

<sup>x/</sup> It is essential to note, that averaging is connected in our case with time and not with physical infinitesimal volume.

5 -function (I). The number of these functions is determined by the number of nonequivalent magnetic ions<sup>x/</sup> in the elementary crystal cell. Each such function  $\vec{M}_j(\vec{r})$  corresponds to different magnetic sublattices in the discrete description.

The functions  $\vec{M}_j(\vec{r})$  in the ground state of magnetic are considered as uniform:  $\vec{M}_{j0}$ . Such uniform "spreading" of magnetic moment over the volume corresponds to completely ordered distribution of the atomic moments over the magnetic sublattice points. The small fluctuations of magnetic moment densities  $\Delta \vec{M}_j = \vec{M}_j(\vec{r}) - \vec{M}_{j0}$ , arise as weak perturbations of the system. For solution of concrete problems of theory one must know the equations of motion for magnetic moment densities (classical or quantum). At first we must find the Hamiltonian  $\mathcal{H}(\vec{M}_j)^{xx/}$  of system as a function of these densities. When determining the Hamiltonian we must use some results of quantum theory of magnetism and also the experimental data for the crystal and magnetic atomic structure of the substances in question. These data may be taken from X-ray and neutron diffraction experiments.

Two types of terms can be expected in this Hamiltonian. First, the terms resulting from the isotropic electrostatic exchange interaction. These quantum terms determine the very existence of bodies with the magnetic ordered structure and also the temperature range of their existence. Then the terms, caused by the anisotropic magnetic (relativistic) interactions. They determine ~~the orientations of the magnetisations~~ the orientations of the magnetisations  $\vec{M}_j$  in the crystal as well as the behaviour of magnetic substances under different external influences (magnetic fields, elastic stresses and so on). The order of magnitude of exchange terms

<sup>x/</sup> The ions, which cannot be transformed into one another in translation  
xx/

For the case of high temperatures it is the thermodynamic potential

( $\sim 10^{-13}$  erg/atom) is much higher than of the "magnetic" ones ( $\sim 10^{-16}$  erg/atom). Therefore the last mentioned may be considered as small perturbations.

3.

Now, let us use the symmetry properties of magnetic crystals to find the phenomenological Hamiltonian  $\mathcal{H}(\vec{M}_j)$ . The fundamental characteristic of magnetic crystal, namely, its magnetic moment, is connected with microcurrents, and therefore with the particle velocities. Therefore in such crystals we must consider also the symmetry as to the change of time sign ( $t \rightarrow -t$  : R -operation<sup>(22/)</sup>). Using this symmetry element we can determine the general criterion for crystals with atomic magnetic structure. If the R-operation is in itself the symmetry element of the crystal (namely, it doesn't change its thermodynamic equilibrium state) then this crystal does not possess any magnetic structure. In fact, by applying R to magnetizations  $\vec{M}_j$ , we change their signs:  $R \cdot \vec{M}_j = -\vec{M}_j$ . Since the crystal state does not change under this operation, the magnetisations also remain invariable ( $\vec{M}_j = -\vec{M}_j$ ). Therefore, there is only one possibility, i.e. that all  $\vec{M}_j = 0$ ; which means that in this case the magnetic structure is absent. If on the contrary, the R-operation isn't the symmetry element of the crystal, then the latter can possess some atomic magnetic structure.

If we know the atomic symmetry of crystal lattice<sup>x/</sup>, we can find  
<sup>x/</sup> Each crystal possesses a definite set of symmetry operations, which transforms it in itself (rotation axes, rotation-reflection axes, screw axes, reflection planes, glide planes, and translations). They form the so called space group of symmetry. The R-operation in itself and in different combinations with all others must be included in this group in the case of crystal without magnetic structure (paramagnetics). Tsvager and Zaitzev<sup>/23/</sup> made a detailed group theoretical investigation of crystal magnetic symmetry.

theoretically all ordered magnetic states, permitted by this symmetry. The crystal lattice symmetry elements must contain the symmetry elements of each such state<sup>x/</sup>. Using these considerations we can find under what conditions among different magnetic states the phase transitions of the first or second order take place<sup>/10,16/</sup>. Sometimes not only one but a great many magnetic states may correspond to the same set of symmetry elements. Then, if the crystal with definite atomic structure admits at least one of these magnetic states, it admits them all as well.

One must know the explicit form of Hamiltonian  $\mathcal{H}(\vec{M}_j)$  and its minimum to find the conditions under which one or the other ordered magnetic state is realized. One should make a number of notes here:

(a)  $\mathcal{H}(\vec{M}_j)$  must remain invariant under all symmetry transformations of crystal lattice. In this case the magnetic structure should not be considered, because for all symmetry operations (including R) of paramagnetic state, all lattice points and axes are transformed into equivalent ones. Therefore each magnetic moment gets in the equivalent lattice point and is oriented along the equivalent axis. Hence there is no reason for the change of  $\mathcal{H}$ .

(b) The translations over distances multiple the period of elementary magnetic cell, leaves all the  $\vec{M}_j$  invariant. Therefore such operations may be excluded. Hence the invariance of  $\mathcal{H}$  only with respect to all rotation and screw axes, reflection, and glide planes, and R-operation is required.

(c) One must know the transformation rules for different  $\vec{M}_c$  (the number of which  $V$  is determined by elementary cell structure) for all symmetry operations. In other words, it is necessary to know the index <sup>x/</sup> The number of magnetic symmetry elements cannot exceed the number of atomic symmetry elements. Therefore the magnetic symmetry group usually forms the subgroup of crystal symmetry group.



permutations  $P(1,2,\dots,\nu)$  of different  $\vec{M}_j$  for all symmetry operations<sup>x/</sup>. The transformation properties of functions  $\vec{M}_j$  include their axial character as well.

(d) The expansion  $\mathcal{H}$  as a series of powers of  $\vec{M}_j$  must contain only even powers because of its invariance with respect to R-operation.

The most general form of expansion of  $\mathcal{H}(\vec{M}_j)$  is given by the formula

$$\mathcal{H} = A_{\alpha\beta}^{\delta\delta'} \frac{M_{j\alpha}}{M_{j0}} \frac{M_{j\beta}}{M_{j0}} + B_{\alpha\beta\gamma\delta}^{\delta\delta'} \frac{a_\gamma}{M_{j0}} \frac{a_\delta}{M_{j0}} \frac{\partial M_{j\alpha}}{\partial z_\gamma} \frac{\partial M_{j\beta}}{\partial z_\delta} +$$

$$+ C_{\alpha\beta\gamma\delta}^{\delta\delta'\delta''\delta'''} \frac{M_{j\alpha}}{M_{j0}} \frac{M_{j\beta}}{M_{j0}} \frac{M_{j\gamma}}{M_{j0}} \frac{M_{j\delta}}{M_{j0}} + \dots - \frac{1}{2} \sum_{\delta\delta'} \vec{M}_j \vec{H}_{j\delta\delta'} - \sum_j \vec{M}_j \vec{H}_j. \quad (2)$$

Here  $A_{\alpha\beta}^{\delta\delta'}$ ,  $B_{\alpha\beta\gamma\delta}^{\delta\delta'}$  and  $C_{\alpha\beta\gamma\delta}^{\delta\delta'\delta''\delta'''}$ , .... are tensors of energy density dimension, independent of  $z_\alpha$ , caused by isotropic electrostatic and anisotropic magnetic interactions;  $a_\gamma$  are length dimension constants of the order of magnitude of lattice parameter.  $\alpha, \beta, \gamma, \delta, \dots = x, y, z$ .  $j, j', \dots = j_1, j_2, \dots, j_\nu$ ;  $\nu$  - the number of magnetic sublattices  $M_{j\alpha}$  -  $\alpha$ -components of vectors  $\vec{M}_j$ . Summation is made over all indices occurring twice in (2). The terms with the space derivatives  $\frac{\partial M_{j\alpha}}{\partial z_\beta}$  describe the deviations from the uniform magnetization in the system.

In (2) we may be satisfied with the terms of second order only in derivatives in the case of longwave oscillations. The ratios  $\frac{M_{j\alpha}}{M_{j0}}$  for each sublattice  $j$  are the directional cosines about the axes  $\alpha$ .

Near the Curie and Néel points the expansion (2) in fact is carried out over small dimensionless parameter  $\frac{M_{j\alpha}}{M_{j0}}$  ( $M_{j0}$  are the maximum

<sup>x/</sup> These permutations are caused by the mutual transitions of magnetic ions from one lattice point into another under the symmetry operations. Dzialoshinsky<sup>/13/</sup> was the first to point to this fact. Earlier it was wrongly thought that different  $\vec{M}_j$  transform only independently from each other under all symmetry operations. Only identical index permutation was taken into account. All other permutations were not considered, and this could lead to loss of important terms in  $\mathcal{H}$ .

ground state magnetisations). The  $\frac{M_{jd}}{M_{j0}}$  may be not small ( $\sim 1$ ) at low temperatures. In this case, therefore, two additional problems of determination of the system ground state ("vacuum") and selection of elementary excitations arise. The magnetic field  $\vec{H}_j^M$  describes the long range part of magnetic interactions, which are not included in tensors A, B, C, .... In general,  $\vec{H}_j^M$  must be found from the Maxwell's equations. We can be satisfied only with magnetostatics equations  $\text{rot } \vec{H}_j^M = 0$  and  $\text{div } \vec{H}_j^M = -4\pi \text{div } \vec{M}_j$ , if we neglect the displacement currents. If we take into account the dipole interaction in the first approximation, we get the expression for the magnetic field intensity

$$\vec{H}_j^M = \nabla \left\{ \int \frac{\text{div } \vec{M}_j(\vec{r}') d\vec{r}'}{|\vec{r} - \vec{r}'|} - \oint \frac{\vec{M}_j(\vec{r}') \vec{n}_0}{|\vec{r} - \vec{r}'|} dS' \right\}. \quad (3)$$

Here in the first term the integration is carried out over the volume and in the second term - over the external surface of specimen ( $\vec{n}_0$  - is the unit length vector normal to this surface). The last term in (2) is the magnetic energy with respect to external magnetic field  $\vec{H}$ .

#### 4.

Now, let us illustrate this theory by two isomorphous rhombohedric crystals  $\text{Cr}_2\text{O}_3$  and  $\alpha\text{-Fe}_2\text{O}_3$  (the space group  $D_{3d}^6$ ). In the elementary cell there are four magnetoactive ions  $\text{Cr}^{3+}$  or  $\text{Fe}^{3+}$  placed along the  $[III]$ -axis (fig. I). According to neutron diffraction data, the magnetic unit cell coincides with the chemical one in these crystals.

The following symmetry operations of paramagnetic state<sup>/26,27/</sup> must be taken into account when we want to examine the possibility of any of magnetic states

$$2 C_3, 3 C_2, \tilde{I}, 2 \tilde{S}_6, 3 \tilde{C}_2, R. \quad (4)$$

Here  $C_3$  are the third order axes parallel to  $[III]$ ;  $C_2$  are the second order axes, perpendicular to  $[III]$ , intersecting  $[III]$  in points "a" (fig. Ib);  $\tilde{I}$  and  $\tilde{S}_6$  are the inversion and the sixth order rotation-reflection axes with respect to points "b" on  $[III]$  axis respectively;

$\tilde{\sigma}_d$  are the glide planes with translation for one half period along  $[III]$  axis and perpendicular to  $U_2$  axes.

Neutron diffraction experiments<sup>/24,25/</sup> proved the existence of two antiferromagnetic structures for  $Cr_2O_3$  (fig.2a) and  $\alpha - Fe_2O_3$  (fig.2b) respectively. The third structure (fig.2c) is possible also. In the fig.2 there are shown only the mutual magnetic moment orientations ~~of antiferromagnetic axis~~ in sites of the cell. Most different orientations of antiferromagnetic axis<sup>x/</sup> are possible as well (for example, along the  $[III]$ , normal to  $[III]$  etc.).

The set of symmetry elements of given type of magnetic structure must transform the unit cell moment distribution in itself. For instance if in the cell there is the resultant moment  $\vec{m} = \sum_j \langle \vec{S}_j \rangle_t \neq 0$ , it must remain invariant under all of these operations. In all the magnetic states of "a" and "b" types, among the allowable operations, there is one complex I.R operation. Applying it to  $\vec{m}$ , we get  $-\vec{m}$ , which gives  $\vec{m} = 0$  because of the invariance requirements. Therefore in "a" and "c" afm structures ferromagnetism is impossible. This agrees with the observed properties of  $Cr_2O_3$ . The symmetry requirements allow three possible variations in the "b"-type states: (I) the afm axis coincides with  $[III]$ ; (II) the afm axis lies in the symmetry plane  $\tilde{\sigma}_d$  (its orientation in  $\tilde{\sigma}_d$  is determined from the minimum value of Hamiltonian); (III) afm axis coincides with one of the  $U_2$  axes, normal to  $[III]$  (fig.3a,b,c).

In case (I) the symmetry elements are given by (4) with the exception of R. Let us assume that there is the resultant moment  $\vec{m} \neq 0$  in the elementary cell. We can resolve  $\vec{m}$  into two components:  $\vec{m}_\parallel$  -parallel to the  $[III]$  axis, and  $\vec{m}_\perp$  -normal to  $[III]$  axis.  $\vec{m}_\parallel$  is invariant <sup>x/</sup>That is the axis along which the magnetizations of sublattices are oriented in both directions.

With respect to  $C_3$  transformation, and  $\tilde{U}_2$  - to  $U_2$ . Therefore the invariance of  $\vec{m} = \vec{m}_1 + \vec{m}_2$  (with respect to  $U_2$  and  $U_2$ ) is possible only if  $\vec{m} = 0$ . Thus ferromagnetism is impossible in afm structure (I). In  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> this is the case at  $T \leq 250^\circ\text{K}$ .

In (II) the symmetry elements are:

$$\tilde{I}, \tilde{G}_d, \text{ and } U_2 \text{ (normal to plane of fig. 3b)} \quad (5)$$

These operations permit rotations of pairs of moments  $\vec{S}_1, \vec{S}_4$  and  $\vec{S}_2, \vec{S}_3$  towards ~~rotation/creates~~ one another from the plane of fig. 3b. This rotation creates a resultant moment  $\vec{m} \neq 0$  along the  $U_2$  axis. Therefore ferromagnetism is possible in this case. In  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> it is observed as "weak" ferromagnetism in the temperature range from  $250^\circ\text{K}$  to  $950^\circ\text{K}$ .

The structure (III) possesses the symmetry elements

$$U_2 R, \tilde{I}, \tilde{G}_d R \quad (6)$$

These elements permit any rotation of moments  $\vec{S}_j$  (at equal angles), causing the appearance of a resultant moment  $\vec{m} \neq 0$ , oriented in  $\tilde{G}_d$  plane. In  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> this structure does not occur.

Now let us consider the isomorphous crystals FeCO<sub>3</sub>, MnCO<sub>3</sub> and CoCO<sub>3</sub>. They are of rhombohedral symmetry type also, but they possess two magnetic ions  $\text{Mn}^{+2}, \text{Fe}^{+2}$  or  $\text{Co}^{+2}$  in the unit cell (fig. 4). The symmetry operations of paramagnetic state are given as before by (4). The  $U_2$  axes intersect  $[\text{III}]$  axis at points "c" (fig. 4). The  $\tilde{I}$  and  $\tilde{S}_6$  must be taken with respect to "d" points. A translation for half a period along the  $[\text{III}]$  axis corresponds to the  $\tilde{G}_d$  glide plane (from lattice point 1 toward 2). Only one afm structure (fig. 2c) with three possible types of orientations of afm axis (fig. 3a, b, c) and fig. 5a, b, c) is possible in this case. It may be proved by analogy with the above that for structure of fig. 5a the ferromagnetism is impossible, while for those of figs 5b, c it is possible. The former occurs in FeCO<sub>3</sub>, and the latter is observed as "weak" ferromagnetism in MnCO<sub>3</sub>.

and  $\text{CoCO}_3$  <sup>/I4/</sup>.

The above mentioned theoretical analysis permits to determine whether any magnetic structures are compatible with the given crystal lattice symmetry or not. In particular we got the general conclusion as to the possibility of co-existence of the whole set of magnetic structures.

### 5.

Further investigations of Hamiltonian (2) are necessary to examine the conditions of occurrence of any of magnetic states. Let us consider  $\text{Fe}_2\text{CO}_3$ ,  $\text{MnCO}_3$ , and  $\text{CoCO}_3$  again <sup>X/</sup>. For the symmetry operations (4) the index transformations of densities  $\vec{M}_1$  and  $\vec{M}_2$  are

$$C_2: 1 \rightarrow 1, 2 \rightarrow 2; \mathcal{U}_2: 1 \rightleftharpoons 2; \tilde{I}, \tilde{S}_2: 1 \rightarrow 1, 2 \rightarrow 2; \tilde{G}_2: 1 \rightleftharpoons 2. \quad (67)$$

Using (6), (7), and axial properties of  $\vec{M}_i$  vectors, we find from (2) (OZ axis // [III]) <sup>XX/</sup>:

$$\begin{aligned} \mathcal{H} = & \frac{A}{2M_0} M_{jd} M_{jd} + \frac{A_1}{M_0} M_{1d} M_{2d} + \frac{B}{2M_0} (M_{1z}^2 + M_{2z}^2) + \frac{B_1}{M_0} M_{1z} M_{2z} + \\ & + \frac{D}{M_0} (M_{1x} M_{2y} - M_{2x} M_{1y}) + \frac{C}{2M_0} (\nabla M_{jd} \nabla M_{jd}) + \frac{C_1}{2M_0} (\nabla M_{1d} \nabla M_{2d}) + \dots \quad (8) \\ & \dots - \frac{1}{2} (\vec{M}_1 + \vec{M}_2) (\vec{H}^M + 2\vec{H}), \quad (\vec{H}^M = \vec{H}_1 + \vec{H}_2) \end{aligned}$$

It is easy to prove, that eqs (8) are invariant for all symmetry operations (3). The terms with A and  $A_1$  describe the exchange interaction inside and between the two magnetic sublattices. The terms with C and  $C_1$  describe the influence of magnetization non-uniformities. The terms with B and  $B_1$  determine the magnetic anisotropy inside and between the sublattices respectively. The term with D of the magnetic nature also, was found first by Dzialoshinsky <sup>/I3/</sup>. This term is the direct

<sup>X/</sup> This is valid for  $\alpha\text{-Fe}_2\text{O}_3$  also.

<sup>XX/</sup> In (8) we neglect all the terms of the order above the second, while in the terms with  $\nabla M_{jd}$  only the terms of isotropic exchange interaction are taken into account.

result of the transformations (7). The term with D determines the "weak" ferromagnetism and influences the resonance frequencies and their dependence from external conditions as well. Though this term is quadratic in  $\vec{M}_j$ , it is different for different "uniaxial" crystals<sup>/28/</sup>. For example, in the case of tetragonal symmetry ( $\text{MnF}_2$ ) this term is not like that in the crystal of rhombohedral system ( $\text{MnCO}_2$ ).

The minimization of Hamiltonian (8) is our next step. Using this operation we can find the relations among the coefficients A, B, .... These relations determine any of the admissible magnetic states of given crystal symmetry. Here we must distinguish two different cases, namely, the cases of high and low temperatures.

## 6.

The formula (8) gives the expansion of potential  $\phi$  in series by  $1/T$  the powers of small parameter  $\frac{M_{jd}}{M_{jo}}$  for high temperatures near the phase transition from paramagnetic to ordered magnetic state. We introduce two new vectors

$$\vec{m} = \vec{M}_1 + \vec{M}_2, \quad \vec{\ell} = \vec{M}_1 - \vec{M}_2, \quad (9)$$

instead of vectors  $\vec{M}_j$  according to Dzialoshinsky<sup>/13/</sup>, to show more clearly the fm and afm components of magnetic structure. The  $\vec{m}$  vector characterizes the fm state while  $\vec{\ell}$  - the afm state. We must also note, that the  $\vec{\ell}$  vector determines the afm axis. Using eqs (9), and omitting the terms with  $(\nabla M_{jd} = \text{div } \vec{M}_j = 0)$  and with the demagnetization effect of sample surfaces, from (8) we find

$$\phi = \frac{a}{2} \ell^2 + \frac{a_1}{2} m^2 + \frac{b}{2} \ell_x^2 + \frac{b_1}{2} m_x^2 + d(\ell_x m_y - \ell_y m_x) + \frac{e}{4} \ell^4 + \frac{f}{4} m^4 - \vec{m} \vec{H} \quad (10)$$

Here coefficients a, b, c, .... are simply expressed by A, B, C, .... from (8). The terms with e and f are exchange invariants of the fourth order with respect to  $\vec{\ell}$  and  $\vec{m}$ . In the "main" exchange terms of eq. (10)  $a$  and  $a_1$  are positive in paramagnetic state. Therefore there is no magnetic order for the minimum value of  $\phi$  ( $\vec{m}=0, \vec{\ell}=0$ ). We shall get a

purely afm structure ( $\vec{m}=0, \vec{l} \neq 0$  at  $\vec{H}=0$ ), if the coefficient "a" changes its sign and  $a_I$  remains  $> 0$  at the point of phase transition between the paramagnetic and ordered states and if we neglect the term with  $d$ . If we take into account the term with  $d$ , then for the minimum value of  $\phi$ , besides the purely afm structure, there exist the structure with  $\vec{m} \neq 0$ , i.e. the "weak" ferromagnetism of relativistic nature. This case is considered in detail in paper<sup>/I3/</sup>.

On the contrary, if at the point of phase transition the coefficient  $a_{nt}$  change its sign and  $a_I > 0$  in both phases, then for the minimum value of  $\phi$  we shall get the fm state ( $\vec{m} \neq 0, \vec{l}=0$ ) of usual exchange type. In this case it is possible to get the admixture of "weak" antiferromagnetism of relativistic type also, if we take into account the term with  $d$ . Indeed, we find the solutions: (I)  $\vec{l}=0, m_x = m_y = 0$  and (II)  $m_z = 0, l_z = 0$  and  $l_x = -\frac{d}{a} m_y, l_y = \frac{d}{a} m_x$ , if we determine the minimum of (IO) with given  $\vec{m}$ . In the state (I) we have ferromagnetism with axis of easy magnetisation along [III] without the admixture of weak antiferromagnetism. In state (II) the axis of easy magnetization lies in (III) plane, but the magnetizations of sublattices are rotated at a small angle towards one another. Therefore the afm components along the afm axis perpendicular to vector  $\vec{m}$  arise. It is necessary to take into account the higher order invariants (with respect to  $m_x$  and  $m_y$ ), which determine the magnetic anisotropy in the (III) plane, to find the orientations of vectors  $\vec{m}$  and  $\vec{l}$  in the same plane.

The investigations of potential (IO) extremums determines the character of phase transitions among the different magnetic states, if we take into account some further terms in the expansion of (IO) (See<sup>/I3/</sup>). Let us note only, that the second order phase transitions demand two conditions for these transitions take place. (I) the symmetry group one of phase must be the subgroup of the other phase.

(2) the expansion of  $\phi$  must not contain any third order terms of small parameter. This is always the case for transition from paramagnetic state to the magnetic ordered one<sup>/10,18/</sup>. In this case we have the second order transition. In other cases more often we have the first order transitions, which don't require the fulfilment of the above mentioned conditions. In particular this ~~case~~ occurs in  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> at 250°K<sup>/13/</sup>.

The invariants, containing elastic stress tensor components, and also their products by the sublattice magnetization components, can be taken into account in the expansion of  $\phi$ . The detailed thermodynamic theory of magnetic ordered medium in the region of phase transition points can be created, if we take into consideration the external magnetic field effect. Essential displacement of phase transition points in magnetic substances can take place under the influence of external magnetic fields and elastic stresses, which can greatly change the magnetic crystal anisotropy.

Many problems of thermodynamic theory of magnetic transitions are investigated in the works of Soviet scientists<sup>/11,12,29,48/</sup> (temperature dependence of spontaneous magnetization, curves of <sup>the</sup> paraprocess, the influence of elastic stress, magnetostriction and spontaneous lattice strain, displacement of transition point, galvanomagnetic and other kinetic effects near the Curie point, etc). The important result of these investigations is the possibility to get the precise theoretical expressions for different physical quantities, as for instance, magnetization, magnetostriction etc. It is possible also to give relations among the phenomenologic constants, which can be proved by experiments.

The terms with the gradients of magnetisation in  $\phi$  can be taken into account in the considered temperature range also. This is important for the calculation of magnetic long range order fluctuations



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near the phase transition points. Here these fluctuations (i.e. magnetic short range order) are relatively very great. This problem was discussed in the work<sup>/II/</sup>.

7.

Now let us consider the applications of phenomenologic theory for low temperatures. First we shall describe briefly the general spin wave method within the limits of this theory.

The ground state and eigenfrequencies spectrum of weakly perturbed states of the system, described by Hamiltonian (2), can be obtained using classic motion equations for magnetizations  $\vec{M}_j(\vec{r})$ , or with the help of the method of second quantization.

The classic motion equations are

$$\dot{\vec{M}}_j = \gamma_j [\vec{M}_j \times \vec{H}_j], \quad (\text{II})$$

Here  $\vec{H}_j$  - effective field, which acts on  $\vec{M}_j$  and is determined from formula<sup>/30/</sup>:

$$\vec{H} = -\frac{\partial \mathcal{H}}{\partial \vec{M}_j} + \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{H}}{\partial \dot{\vec{M}}_j} \right). \quad (\text{I2})$$

The point on left hand side of (II) means the time differentiation;  $\gamma_j$  - magnetomechanic ratio (or more precisely, the factor of spectroscopic splitting) for  $j$  sublattice,  $\gamma_j = g_j \beta / \hbar$ , where  $\beta$  - Bohr's magneton,  $2\pi \hbar$  - Plank's constant and  $g_j$  - Lande factor.

The equilibrium positions of vectors  $\vec{M}_{j0}$  are found from conditions  $\dot{\vec{M}}_j = 0$  or from minimum of Hamiltonian (2). Now let us suppose  $\vec{M}_j = \vec{M}_{j0} + \Delta \vec{M}_j$  and linearize the system of eqs (II) with respect to deflections  $\Delta \vec{M}_j$ . The solution of this system usually is carried out in the form  $\Delta \vec{M}_j \sim \exp[i(\omega t + \vec{K} \cdot \vec{r})]$ , where  $\omega$  - frequency, and  $\vec{K}$  - vector of quasimomentum. The eigenfrequencies  $\omega$  and the dispersion relations for them  $\omega(K)$  are found if we take the system determinant as being equal to zero.

The method of second quantization, which is the generalization of the well known Holstein-Primakoff's method<sup>/31/</sup>, is often, however,

more simple to use. This method has a number of advantages over the classic method, because the zero point energy is taken into account. The relaxation processes may be considered. This method is also convenient for its "automatic" character. Namely, to get the dispersion relations<sup>/32/</sup>, it is enough to find the quadratic form of second quantization operators.

Using the method of second quantization, the classic pseudovectors  $\vec{M}_j$  must be replaced by the operators  $\hat{M}_j(\vec{r})$ , which satisfy the following commutation rules<sup>/33/</sup>:

$$\hat{M}_{j\alpha}(\vec{r})\hat{M}_{j'\alpha'}(\vec{r}') - \hat{M}_{j'\alpha'}(\vec{r}')\hat{M}_{j\alpha}(\vec{r}) = i\mu_j \hat{M}_{j\alpha} \delta_{\alpha\alpha'} \delta(\vec{r}-\vec{r}') \text{ etc.} \quad (I3)$$

where  $\mu_j = g_j \beta$ . The second quantization operators  $\hat{b}_z^{\dagger}$  and  $\hat{b}_z$ , given by formula<sup>/31,34/</sup>

$$\begin{aligned} \hat{M}_{jx_j} &= (\frac{1}{2}\mu_j M_{j0})^{1/2} (\hat{f}_j \hat{b}_z^{\dagger} + \hat{b}_z^{\dagger} \hat{f}_j), \\ \hat{M}_{jy_j} &= (\frac{1}{2}\mu_j M_{j0})^{1/2} (\hat{f}_j \hat{b}_z^{\dagger} - \hat{b}_z^{\dagger} \hat{f}_j), \\ \hat{M}_{jz_j} &= M_{j0} - \mu_j \hat{b}_z^{\dagger} \hat{b}_z; \quad (\hat{f}_j = [1 - \mu_j \hat{b}_z^{\dagger} \hat{b}_z / 2M_{j0}]^{1/2}) \end{aligned} \quad (I4)$$

can be introduced, if the own coordinate system  $Ox_j Y_j Z_j$  is chosen for each sublattice  $j$ , so that  $OZ_j$  axis is parallel to quantization axis of  $\vec{M}_j$  vector (i.e. along classic  $\vec{M}_{j0}$  vector). The mean values of magnetization deflections are relatively small:  $\langle \mu_j \hat{b}_z^{\dagger} \hat{b}_z \rangle_{av} = \langle \Delta M_{jz_j} \rangle_{av} \ll M_{j0}$ , because of the initial assumption that the system perturbations are weak. Inserting (I4) in (I3), it is easy to see, that operators  $\hat{b}_z^{\dagger}$  and  $\hat{b}_z$  agree with the Bose commutation relations

$$\hat{b}_z^{\dagger} \hat{b}_z^{\dagger} - \hat{b}_z^{\dagger} \hat{b}_z^{\dagger} = \delta_{jj'} \delta(\vec{r}-\vec{r}'). \quad (I5)$$

It is necessary to pass to one common coordinate system  $OXYZ$ , connected with the crystal axes. This transition is made by using the table of directional cosines  $a_{\alpha\beta_j}$ :

$$\hat{M}_{j\alpha} = \sum_{\beta_j} a_{\alpha\beta_j} \hat{M}_{j\beta_j}. \quad (I6)$$

The whole Hamiltonian of the system can be represented as a series

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3 + \dots \quad (I7)$$

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by introducing the Fourier-components  $b_k^+$  and  $b_k^-$  of  $b_k^+$  and  $b_k^-$  respectively. Here  $\mathcal{H}_n$  is the  $n$ -order term with respect to operators  $b_k^+$  and  $b_k^-$ .

The ground state, or the equilibrium values of  $a_{\alpha\beta}^0$  at  $T=0^\circ\text{K}$ , can be found by minimization of the zero order term  $\mathcal{H}_0$  (independent from  $b_k^+$  and  $b_k^-$ ) with respect to directional cosines. The first order term  $\mathcal{H}_1$  in (I7) is identically equal to zero for  $a_{\alpha\beta}^0$ . Therefore the ground state can be also found from the condition  $\mathcal{H}_1 = 0$ . The term  $\mathcal{H}_2$  can be presented as the sum of individual elementary excitations - spin waves, after reducing diagonal form. This term determines the magnetic selfoscillation spectrum in first approximation. The higher order terms in (I7) ( $\mathcal{H}_3$  etc) describe the "collision" processes among spin waves (ferromagnons, antiferromagnons). The different kinetic phenomena in magnetics can be investigated, using this theory terms.

8.

The case of uniaxial ferromagnetics with the easiest magnetization axis along OZ, influenced by external field  $\vec{H}$  of arbitrary magnitude and orientation, may be taken as a concrete example. This problem was investigated in a number of works<sup>/19,31/</sup> (for the case  $\vec{M}_0 \parallel \vec{H}$ ). It is known, that for  $\vec{H} \perp \text{OZ}$  there exists a critical field  $\vec{H}_A$ , which is determined by magnetic anisotropy energy, and by which the rotation of vector  $\vec{M}$  towards the field is accomplished. The dependence of field  $H_A (= H_A(T))$  from  $T$  was not taken into account<sup>/19,31/</sup>, however, when calculating the temperature change of the magnetization curves (at  $H < H_A$ ). This dependence  $H_A(T)$  is due to temperature change of the equilibrium position of  $\vec{M}_0$ , i.e. from the temperature dependence of anisotropy constant. Therefore, the following expressions for functions  $M_n(T)$  were found<sup>/36/</sup> for  $\vec{H} \perp \text{OZ}$ .

$$M_H(T) = M_0 \frac{H}{H_A} \left[ 1 + 0.05 \frac{M}{M_0} \left( \frac{H}{H_A} \right)^{3/2} \right]; \quad (H \leq H_A^T);$$

$$M_H(T) = M_0 \left[ 1 - 0.05 \frac{M}{M_0} \left( \frac{H}{H_A} \right)^{3/2} \right]; \quad (H \geq H_A^T);$$

$$H_A^T = H_A \left[ 1 - 0.12 \frac{M}{M_0} \left( \frac{H}{H_A} \right)^{3/2} \right];$$

(I-exchange interaction parameter).

Therefore the theory predicts the increase of magnetization (at  $H \leq H_A^T$ ) with the rise of temperature in the unsaturated ferromagnetics. Such effect is observed in common polycrystalline samples as well. However the rise of magnetization in the latter case is due not only to facilitation of rotation processes, but also of the displacement processes of domain boundaries in ferromagnetics.

9.

Now, let us consider the ferromagnetics with two magnetic sublattices, the magnetizations of which in the ground state are antiparallel and are not equal to each other:  $M_{10} \neq M_{20}$ . The resultant magnetization is not zero and is equal to  $M_{10} - M_{20}$  and directed along the field  $\vec{H}$  at  $T=0^{\text{K}}$ . The spin wave spectrum is given by

$$\varepsilon(k)^{(1,2)} = \left[ (\varepsilon_1 + \varepsilon_2)^2 - \varepsilon_{12}^2 \right]^{1/2} \pm (\varepsilon_1 - \varepsilon_2) \quad (18)$$

if we do not consider the magnetic terms in Hamiltonian (isotropic ferromagnetics). Here

$$2 \varepsilon_j = \frac{M_j}{M_{j0}} (A_{12} + C_{jj} k^2) - (-1)^j M_j H, \quad (j=1,2), \quad (19)$$

$$\varepsilon_{12} = \left( \frac{M_1 M_2}{M_{10} M_{20}} \right)^{1/2} (A_{12} + C_{12} k^2)$$

$A_{12}, C_{jj}$  - phenomenological constants of exchange interaction. The ferromagnetic and exchange resonance frequencies may be found from (18) at  $k=0$ . One can easily see, that the spin wave energy (18) is in linear dependence from  $k$  (at  $H=0$ ) under the condition

$$\frac{M_{10}}{M_1} = \frac{M_{20}}{M_2} \quad (20)$$

This fact is characteristic for the afm state. Formula (20) permits the existence of the resultant magnetization, i.e. ferromagnetism

as well. Indeed, from (20) it follows:

$$M_{so} = M_{10} - M_{20} = \frac{\Delta\mu}{\mu} M_0 \quad (21)$$

where  $\Delta\mu = \frac{1}{2}(\mu_1 - \mu_2)$ ,  $\mu = \frac{1}{2}(\mu_1 + \mu_2)$  and  $M_0 = \frac{1}{2}(M_{10} + M_{20})$ . Therefore, on the one hand the compensated spin antiferromagnetism occurs, because  $N_1 S_1 = N_2 S_2$  according to (20) ( $N_1$  and  $S_1$  - numbers of magnetic ions and their spins in sublattices, respectively). On the other hand the non-compensated "orbital" ferromagnetism appears, caused by the different g-factors of ions in different sublattices. This may result from the existence of different ions with  $g_1 \neq g_2$  in the sublattices. The  $g_i$ -factors nevertheless can be different in the case of identical ions, if these ions occupy nonequivalent sites with different type of nearest neighbourhood in the lattice. We obtain

$$\varepsilon(k)^{(1,2)} = \left\{ \hbar^2 k^2 + \mu^2 \left[ \frac{\Delta\mu}{\mu} H_E H + \left( \frac{\Delta\mu}{\mu} \right)^2 H^2 \right] \pm \mu H \right\} \quad (22)$$

for the spin wave energy of this "orbital" ferromagnetism (using eqs. (18), (19) and (20) at  $C_{11} = C_{22}$ ). Here  $H \leq \frac{\Delta\mu}{\mu} H_E$ , and  $H_E$  - effective exchange field, I-constant defined by exchange coefficients  $A_{12}$  and  $C_{ij}$  from eq. (19). The temperature and field dependence of magnetization can be calculated by using the spectrum (22). This dependence is (under condition  $\chi T \gg [\Delta\mu H_E - \mu H]^2$ )

$$M(T, H) = M_s(T) + \chi(T) \cdot H \quad (23)$$

Here  $M_s$  - spontaneous magnetization, which is equal

$$M_s(T) = \frac{\Delta\mu}{\mu} M_0 \left[ 1 - \frac{\mu^2 H_E (\chi T)^2}{12 M_0 T^3} \right] \quad (24)$$

and  $\chi$  - susceptibility (analogous of  $\chi_H$  in antiferromagnetics), which is equal to

$$\chi(T) = \frac{\mu^2 (\chi T)^2}{3 T^3} \quad (25)$$

Therefore, the spontaneous magnetization in this case follows the  $T^2$  law, and not the  $T^{3/2}$  law as in ordinary ferro- and ferrimagnetics<sup>x/</sup>.

<sup>x/</sup>The  $T^2$  law for spontaneous magnetization of ferrites, discovered in the work<sup>/35/</sup> is valid in that case ( $N_1 S_1 = N_2 S_2$  and  $g_1 \neq g_2$ ).

It should be noted, that the part of magnetization, which depends upon the temperature

$$\Delta M_T = - \frac{\mu^2 (\chi T)^2}{12 T^3} \left( \frac{\Delta \mu}{\mu} H_E - 4H \right)$$

changes its sign at the field  $H = \frac{1}{4} \frac{\Delta \mu}{\mu} H_E$ .

10.

Now, let us consider the case of antiferromagnetism. The atomic theory of antiferromagnetism is based upon the supposition, that there exists an ordered ("checkered") alternation of "right" and "left" ion spin components in the crystal lattice points. The entirely definite spin orientation is ascribed to each ion. This is in contradiction with quantum mechanics, because the individual ions cannot possess definite spin components, owing to exchange interaction. This difficulty does not exist in phenomenological theory, in which one uses only the symmetry properties of afm crystals.

Only one natural afm axis (trigonal, tetragonal or hexagonal) exists in the uniaxial afm crystals. If we omit the higher order terms, which describe the anisotropy in the base plane, then the Hamiltonian will be the same for all these crystals, with the exception of the terms with D (see eq. (8)), which may be different. For instance, this term has the form  $\frac{D}{M_0^2} (M_{ix} M_{iy} - M_{ix} M_{iy})$  in tetragonal crystals (MnF<sub>2</sub> type and the like). This expression is not the same as the term with D in (8) for the rhombohedral crystal. The terms with D must be taken into account in some cases, though they have not great influence upon the general character of magnetic properties of uniaxial antiferromagnetics. For the details of calculations the readers are referred to the original papers<sup>/36,37,38/</sup>. Here we shall mention briefly only some results, connected with the consideration of the term with D. Let the external field H be parallel to OZ axis. Then the afm axis  $\vec{\Delta}$  will be normal to OZ axis at the fields greater than the threshold value ( $H > H_0$ ). If the term with D is considered, there appear the magne-

tization components  $M_{\perp} \sim 2M_0 \frac{H_D}{H}$  perpendicular to OZ axis (and  $\vec{H}$ ), besides the z-components of magnetization  $M_z = M_{1z} + M_{2z} = 2M_0 \frac{H}{H_E}$  proportional to external field  $\vec{H}$ . Here  $H_D = \frac{D}{M_0}$  - is the "Dzialoshinsky field". The magnetization  $\vec{M}_1$  in the first approximation is independent from the external field and is of the same relativistic nature as the "weak" ferromagnetism in  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> (see n°.6). Let us note, that in general  $M_{\perp} \ll M_{\parallel}$ , since  $H_0 \gg H_D$ , because  $H_0$  is determined by the mean geometrical value of exchange and magnetic energies and  $H_D$  is determined only by magnetic energy. The spin wave spectrum and the afm resonance frequencies are determined in the paper<sup>/36/</sup>. The splitting in this spin wave spectrum remains even at  $H=0$ , if we take into account the long range dipol-dipol magnetic interaction. However, this splitting is relatively small, because the constant "magnetic addition" is always small in comparison with the exchange interaction term.

The term with D influences the resonance frequencies in a different ways for rhombohedral, and tetragonal crystals. In the former case  $h\nu = \mu [H_0^2 - H_D^2]^{1/2} \mu H$ , and in the latter  $h\nu = \mu [(H_0 \pm H)^2 - H_D^2]^{1/2}$ . The energy gaps are different in the various branches of antiferromagnon spectrum. The resonance frequencies have different dependences from the magnetic field in the state  $\vec{A} \perp \vec{H}$  at  $\vec{H} \parallel$  OZ and  $\vec{H} \perp$  OZ.

The temperature dependences of magnetic susceptibility~~ies~~ and spin heat capacity were calculated in the paper<sup>/36/</sup> for various orientations and magnitudes of magnetic fields, using spin wave spectrum. The anisotropy of these dependences was determined in the paper<sup>/39/</sup>. Some of these deductions are in good qualitative agreement with the results of the experiments of Handel and other<sup>/40/</sup>. The theoretical conclusion as to the relation between the parts (dependent on temperature) of perpendicular and parallel susceptibilities of uniaxial antiferromagnetics is of interest. This dependence has

the following form  $\Delta \chi_z^L = -\Delta \chi_z'' \approx \alpha T^2$  <sup>x/</sup>. The present experimental data, unfortunately, are insufficient for verification of this quantitative conclusion of spin-wave theory.

## II.

Finally, let us cite the results of the calculations of spin-wave spectrum, of resonance frequencies, and of temperature dependence of magnetization for "weak" ferromagnetics, like  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> and MnCO<sub>3</sub> for low temperatures <sup>/41/</sup>. Here two branches of spin-wave spectrum were found also

$$\varepsilon(k)^{(1)} = [M^2 (H + H_D) H + I^2 k^2]^{1/2}, \quad (26)$$

$$\varepsilon(k)^{(2)} = \{M^2 [H_0^2 + H_D (H_D + H)] + I^2 k^2\}^{1/2}. \quad (27)$$

Here all symbols are the same as in reference 9. ~~Here all symbols are the same as in reference 9.~~ At  $k=0$  the resonance frequencies are

$$h\nu_1 = M [(H + H_D) H]^{1/2}, \quad (28)$$

$$h\nu_2 = M [H_0^2 + H_D (H_D + H)]^{1/2}. \quad (29)$$

The frequency  $\nu_1$  in (28) corresponds to the usual fm resonance, when the easy magnetization axis of uniaxial crystal lies in the base plane. The formula (28) in contrast with that of Kittel <sup>/49/</sup> contains the Dzialoshinsky field  $H_D$  instead of the magnetic (uniaxial) anisotropy field  $H_A$ . To explain the experimental data for fm resonance in  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> <sup>/42/</sup> it is necessary to take into account, perhaps, the magnetic anisotropy in the base plane <sup>/43/</sup>. The second resonance frequency (29) lies in the shorter wave-length region. This frequency corresponds to the usual afm resonance. As far as we know, this resonance was not observed in  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> as yet.

For the temperature interval  $M H_0 \ll \alpha T \ll \alpha T_N$  ( $T_N$  - Neél point), using the spectrum (26) and (27), we find:

$$M(T, H) = M_s(T) + \chi(T) \cdot H = \chi(T) (H_D + H), \quad (30)$$

<sup>x/</sup> Here  $\Delta \chi = \chi(T) - \chi(0)$ ; the lower index indicates the orientation of magnetizing field  $\vec{H}$  and the upper index - the mutual orientation



Here  $M_s$  -the spontaneous magnetization, which is equal to

$$M_s(T) = 2M_0 \frac{H_D}{H_E} (1 - \alpha T^2) \quad (31)$$

and  $\chi$  -susceptibility (analogous to  $\chi_1$  of antiferromagnetics), which is

$$\chi(T) = \frac{2M_0}{H_E} (1 - \alpha T^2) \quad (32)$$

$\alpha$  -constant. The coefficient  $\alpha$  in (31) must be substituted by  $\frac{1}{2}\alpha$ , if the threshold field  $H_0$  is so great, that  $\mu H_0 \gg \alpha T \gg \mu H_1, \mu H_D$ .

Thus the spontaneous moment of "weak" ferromagnetism follows the  $T^2$ -law instead of the  $T^{3/2}$  law like the "orbital" ferromagnetism (see <sup>reference</sup> 9). Borovic-Romanov and Orlova<sup>14</sup> got the two-term formula analogous to (31) for  $MnCO_3$  and  $CoCO_3$ . Unfortunately, one can speak only about the qualitative agreement between theory and experiment at present. Measurements in the lower temperature range for more detailed verification of this theory should be made. However this checking is reasonable only if there is no phase transition from the "weak" ferromagnetic state to pure antiferromagnetic one (like in  $\alpha$ - $Fe_2O_3$ ) in these substances.

It is interesting to note, that "the Dzialoshinsky field"  $H_D$ , which is equal to  $\frac{M_s}{\chi}$ , may be independently determined, measuring  $M_s$  and  $\chi$  and comparing the above value of  $H_D$  with  $H_D$  found from resonance experiments.

The comparison of formula (32) for susceptibility of weak ferromagnetics with the respective formula for  $\chi_1$ , the usual in case of antiferromagnetics in  $\vec{A} \perp \vec{H}$  state, shows that they are similar. This fact may help to distinguish easily the weak ferromagnetism of the "transverse" type (see <sup>reference</sup> 9) from the "longitudinal orbital" one. The linear dispersion law for spin-wave energy is realized for all "weak" ferromagnetics as well as for the antiferromagnetics:  $\xi(k) = \hbar\omega$  (at  $\alpha T \gg \Delta E$ , where  $\Delta E$  -energy gap for the spin-wave

-26-

excitation). Therefore the spin heat capacity must be proportional to  $T^3$ . The formulae for this heat capacity are the same as those found in the case of antiferromagnetism<sup>/39/</sup>.

## I2.

In conclusion of our report we must note, that the phenomenological theory can be applied with considerable success not only to a great number of problems of magnetic phenomena, but to the solid state physics as a whole, as well. We cannot mention, of course, all these problems in this report: the phenomenological treatments the problems of interaction of magnetic and elastic properties of ferro- and antiferromagnetics<sup>/44,45/</sup>, the problems of interaction between spin-waves and conducting electrons in transition metals (s-d - exchange model<sup>/46,34/</sup>), etc, may be mentioned here. Great advantages of strict thermodynamical treatment for explanation of many important problems of magnetic phenomena from all the above stated are now quite evident

References.

1. P. Weiss, Journ. de phys. et radium, 6, 661 (1907).
2. J. H. Van Vleck, Journ. Chem. Phys. 9, 85 (1941).
3. L. Néel, 3, 137 (1948).
4. E. Ising, Z. Phys. 31, 253 (1925).
5. J. I. Frenkel, Z. Physik, 49, 31 (1928).
6. W. Heisenberg, Z. Physik, 49, 619 (1928).
7. K. Feierls, Proc. Camb. Phil. Soc. 32, 477 (1936).
8. L. S. Stilbance, JETP, 9, 432 (1939).
9. S. V. Vonsovsky, DAN USSR, 27, 550 (1940).
10. L. D. Landau, JETP, 7, 19 (1937).
11. S. V. Vonsovsky, Bull. Ac. Sc. USSR. ser. phys. II, 485 (1947); S. V. Vonsovsky and J. S. Shur "Ferromagnetism", Moscow (1948).
12. V. L. Ginsburg, JETP, 17, 833 (1947).
13. I. E. Dzialoshinsky, JETP, 32, 1547 (1957).
14. A. S. Borovic-Romanov and M. P. Orlova, JETP, 31, 579 (1956).
15. L. D. Landau, JETP 7, 1232 (1937).
16. L. D. Landau and E. M. Lifshits "Statistical Physics", Moscow (1951).
17. B. M. Zaitzev, JETP, 34, 1302 (1958).
18. E. M. Lifshits, Journ. Phys. USSR, 8, 61 (1942); L. D. Landau and E. M. Lifshits Sov. Phys. 8, 153 (1935).
19. C. Herring and Ch. Kittel, Phys. Rev. 81, 869 (1950).
20. G. Heller and H. Kramers, Proc. Roy. Acad. Amsterdam, 37, 378 (1934).
21. W. Böhring, Zs. f. Phys. 124, 501 (1947).
22. L. D. Landau and E. M. Lifshits "Electrodynamics of continuous mediums" Moscow (1957).
23. B. A. Tavger and V. M. Zaitzev, JETP, 30, 564 (1956).
24. C. G. Shull, W. A. Shtrauser, and K. O. Wollan, Phys. rev. 83, 333 (1951).
25. B. N. Brockhause, Journ. Chem. Phys. 21, 961 (1953).
26. R. W. C

27. Internationale Tabellen zur Bestimmung von Kristallstrukturen,  
Bd. I (1935).
28. I. E. Dzialoshinsky, JETP, 33, 1454 (1957).
29. K. F. Belov, Usp. Phys. Nauk, 65, 968 (1951).
30. J. R. Mackdonald, Proc. Phys. Soc. 64, 968 (1951).
31. T. Holshein and H. Primakoff, Phys. Rev. 58, 1098 (1940).
32. N. N. Bogolubov "Lectures of quantum statistics" Kiev (1949).
33. Ch. Kittel and E. Abrahams, Revs. Mod. Phys. 25, 233 (1953).
34. E. A. Turov and V. G. Shavrov, Papers of Inst. Phys. Met. Ac. Sc. USSR,  
Sverdlovsk (1958).
35. S. V. Vonsovsky and U. M. Seidiv, Bull. Ac. Sc. <sup>USSR</sup> Ser. phys. 18, 319 (1954).
36. E. A. Turov and U. F. Irchin, Bull. Ac. Sc. <sup>USSR</sup> Ser. phys. 22, N° 10 (1958).
37. M. I. Kaganov and V. M. Tsukernik, JETP, 34, 106 (1958).
38. L. Néel, Ann. de phys. 8, 232 (1936); Bull. Ac. Sc. <sup>USSR</sup> Ser. phys. ~~22~~ 21, 336 (1957).
39. E. A. Turov, JETP, 34, 1009 (1958).
40. J. Van den Handel, H. M. Gijeman, N. J. Foulis, Physica, 18, 862 (1952).
41. E. A. Turov, JETP, (to be published)
42. H. Kumagai, H. Obe, K. Ono, J. Hayashi, J. Shimada, K. Ivanaga, Phys. Rev. 32,  
(1955).
43. M. Shimizu, Journ. Phys. Soc. Japan, 11, 1078 (1956).
44. E. A. Turov and U. F. Irchin, JNM, 3, 15 (1956).
45. M. I. Kaganov and V. M. Tsukernik, JETP, 34, 1610 (1958); A. I. Achieser, V. G.  
Bariachtar, S. V. Peletminsky, JETP, 34, 228 (1958); A. I. Achieser and  
L. A. Shishkin, JETP, 34, 1267 (1958).
46. S. V. Vonsovsky, Journ. Phys. USSR, 10, 468 (1946); S. V. Vonsovsky and E. A.  
Turov, JETP, 24, 419 (1953).
47. R. M. Bozorth, V. Kramers and J. F. Remeika, Phys. Rev. Letters, 1, 3 (1958).
48. K. B. Vlasov and B. H. Ishanchametov, JETP, 27, 75 (1954).
49. Ch. Kittel, Phys. Rev. 73, 155 (1948).

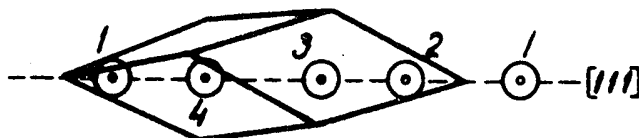


Fig.1a. The unit cell of rhombohedral crystal ( $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>, Cr<sub>2</sub>O<sub>3</sub>)†

⊙ ⊙ ⊙ -ions Fe<sup>+3</sup> or Cr<sup>+3</sup>

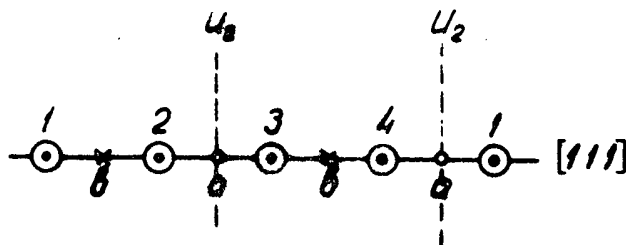


Fig.1b. "a" - the points of intersection of U<sub>2</sub> axes with [111] ; "b" - the points of inversion and of intersection of reflection plane for S<sub>6</sub> axes with [111] .

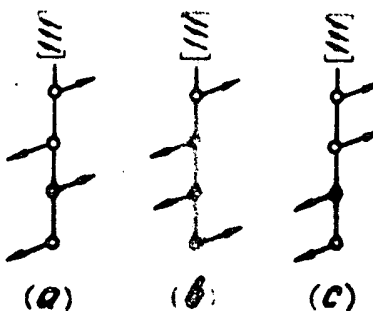


Fig.2. Possible types of antiferromagnetic structures: (a) - Cr<sub>2</sub>O<sub>3</sub>, (b) - Fe<sub>2</sub>O<sub>3</sub>, (c) - unknown.

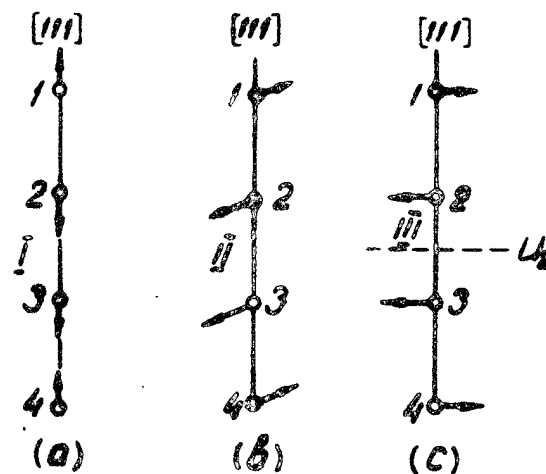


Fig.3. Three possible types of orientations of afm axis in structure of fig.2b ( $\alpha\text{-Fe}_2\text{O}_3$ ).

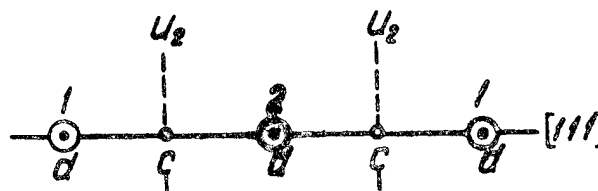


Fig.4. Order of magnetic ions (1,2) along  $[111]$  in crystal  $\text{MnCO}_3, \text{FeCO}_3, \text{CoCO}_3$ .  
 "c" - the points of intersection of  $U_2$  axes with  $[111]$ . "d" - the points of inversion and of intersection of reflection plane for  $S_6$  axes with  $[111]$ .

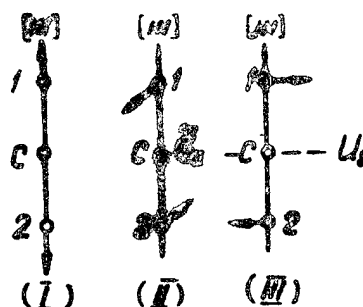


Fig.5. Three possible types of orientations of afm axis in afm structure of  $\text{MnCO}_3$  and  $\text{CoCO}_3$ .